RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2015

THIRD YEAR

Date : 15/12/2015 **PHYSICS (Honours)**

Time : 11 am – 1 pm **Paper : V [Group – A]** Full Marks : 50

[Use a separate Answer Book for each unit]

Unit – I

Answer **any three** questions

1. (i) Explain the nature of the constraints in the following problems of
   a) a particle moving on the surface of a sphere with variable radius,
   b) a pendulum with variable length,
   c) a disc rolling on an inclined plane without slipping.
   (ii) Solve the problem of the motion of two masses connected by an inextensible string moving over a weightless pulley by D'Alembert's principle. (5+5)
2. (i) Solve from Lagrange's equation the problem of the oscillation of a pendulum in a lift moving downwards with an acceleration *f*.
   (ii) From the argument of homogeneity and isotropy of space and homogeneity of time for an inertial frame, show that the Lagrangian of a free particle can only be of the form $\mathrm{L} = \mathrm{L}(\vec{\mathrm{v}} \cdot \vec{\mathrm{v}})$ where $\vec{\mathrm{v}}$ is the velocity vector of the particle. Hence show that $\vec{\mathrm{v}}$ is constant. (5+5)
3. (i) Arrive at the expression for the Hamiltonian of a particle moving in a central potential. Find out the conserved quantity, if any, and deduce the canonical equations of motion. (2+2+2)
   (ii) Define the Poisson bracket of two dynamical variables F, G which depend on the co-ordinates q_α, momenta p_α and the time *t*, Show that
   a) $[F_1 + F_2, G]_{PB} = [F_1, G]_{PB} + [F_2, G]_{PB}$ (2)
   b) $[F, p_\alpha]_{PB} = \dfrac{\partial F}{\partial q_\alpha}$ (2)
4. (i) Explain the terms 'body cone' and 'space cone' in connection with the motion of a free symmetrical top. (6)
   (ii) A rigid body is rotating freely about the third principal axis about which the moment of inertia is I_3. The body is given a small displacement. Show by using the Euler's equations, that the rotation about the third axis is stable if I_3 is either the largest or the smallest of the three principal moments of inertia but not if it is the middle one. (4)
5. (i) Consider the small oscillations in one dimension of a particle near the stable equilibrium which is taken to be the origin $x = 0$. Show that any arbitrary conservative potential will lead to a simple harmonic motion about the equilibrium point which is the minimum for the potential. (4)
   (ii) Suppose a system has two degrees of freedom with generalized co-ordinates q_1, q_2. The kinetic energy of the system is given by $T = \dfrac{\dot{q}_1^2 + \dot{q}_2^2}{2}$. For small q_1 and q_2 the potential energy

of the system is given by $V = \frac{1}{2}k_{11}q_1^2 + k_{12}q_1q_2 + \frac{1}{2}k_{22}q_2^2$. Set up the equations of motion and find the normal modes and their frequencies. (6)

Unit – II

Answer **any two** questions

6. (i) Obtain Einstein's velocity addition formulae. (5)
   (ii) An astronaut is travelling in a space vehicle with a velocity 0.6c relative to the earth transverse to the line connecting the earth to the astronaut. The astronaut measures his pulse rate as 75 per minute. What is the pulse rate as observed from the earth? (5)
7. (i) Obtain the formula for relativistic Doppler effect. How does it differ from the acoustic Doppler effect? (5)
   (ii) Light of wave-length 600 nm is incident normally on a mirror which is receding away from the source with a velocity $3x10^4$ m/sec. Calculate the change in wavelength on reflection from the mirror. (5)
8. (i) Show how the mass of a particle depends on its velocity. (5)
   (ii) Starting from the expression of relativistic momentum (p) and energy (E) of a particle show that $E^2 = c^2p^2 + m_0^2c^4$ where the symbols have the usual meaning. (5)
9. (i) Discuss stellar aberration and explain how it can be explained by the theory of relativity. (5)
   (ii) Find the threshold energy in the laboratory system for the production of a neutral pion through the reaction $p + p \rightarrow p + p + \pi^0$.

   Rest masses of proton and the pion are respectively 940 MeV and 140 MeV in energy units. (5)

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